



Class 11 Maths || Chapter 8 : Binomial Theorem || Class Notes

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Class-11-Maths chapter 8 Binomial Theorem

Binomial Expansions (2 terms) $(a+b)$

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1.a + 1.b$$

$$(a+b)^2 = 1.a^2 + 2.ab + 1.b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^5 = 1.a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

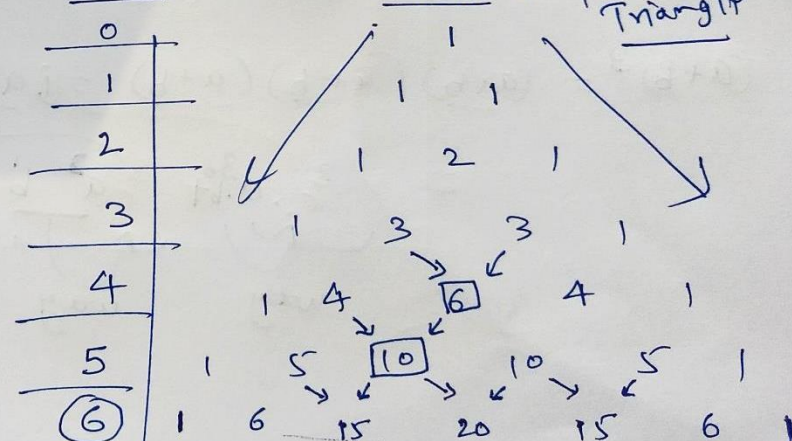
$$(a+b)^6 = 1.a^6 + 6.a^5b + 15.a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6.ab^5 + 1.b^6$$

Variables.

Term = Coefficient x Variable

coefficients.

Powers.



YouTube पर इस Chapter के FREE Videos देखिए (Playlist की Link नीचे है)

<https://www.youtube.com/watch?v=tB4dLvpXMf4&list=PLPFrn0ppwwkQHXLsgBJou1tG4uffnT1h4>

For Better Clarity.

$$(a+b)^3 = \underbrace{(a+b)}_{(a+)(+b)} \underbrace{(a+b)}_{(a+)(+b)} \underbrace{(a+b)}_{(a+)(+b)} = 1 \cdot \underline{a^3} + 3 \underline{a^2 b} + 3 \underline{a b^2} + 1 \cdot \underline{b^3}$$

$$\xrightarrow{\quad} a^3 + \underline{a^2 b} + \underline{a^2 b} + \underline{a^2 b} + \underline{a b^2} + \underline{a b^2} + \underline{a b^2} + \underline{b^3}$$

$$\xrightarrow{\quad} \underbrace{(a+)(a+)(+b)}_{(a+)(+b)(+b)} \quad \underbrace{(a+)(+b)(+b)}_{(a+)(+b)(+b)} \quad \underbrace{(a+)(+b)(+b)}_{(a+)(+b)(+b)}$$

$$\xrightarrow{\quad} \underbrace{(a+)(-+b)(-+b)}_{(a+)(-+b)(-+b)} \quad \underbrace{(a+)(-+b)(-+b)}_{(a+)(-+b)(-+b)} \quad \underbrace{(a+)(-+b)(-+b)}_{(a+)(-+b)(-+b)}$$

$\frac{n!}{r!b!} \rightarrow$ P&C Chapter 7

$${}^n C_r = \underline{\text{select 'r' out of 'n'}} = \frac{n!}{(n-r)!r!}$$

$$(a+b)^3 = \underline{(a+b)} \underline{(a+b)} \underline{(a+b)} = \underline{1} \underline{a^3} + \underline{3} \underline{a^2 b} + \underline{3} \underline{a b^2} + \underline{1} \underline{b^3}$$

$$= \underbrace{{}^3 C_0 \cdot a^3 \cdot b^0}_{\text{way}} + \underbrace{{}^3 C_1 \cdot a^2 \cdot b^1}_{\text{way}} + \underbrace{{}^3 C_2 \cdot a \cdot b^2}_{\text{way}} + \underbrace{{}^3 C_3 \cdot b^3 \cdot a^0}_{\text{way}}$$

Binomial Theorem (for Binomial Expansion)

$n = \text{whole No.}$

$$(a+b)^n = \underbrace{{}^nC_0 a^n b^0} + \underbrace{{}^nC_1 a^{n-1} b^1} + \underbrace{{}^nC_2 a^{n-2} b^2} + \underbrace{{}^nC_3 a^{n-3} b^3} + \dots + \underbrace{{}^nC_{n-1} a^1 b^{n-1}} + \underbrace{{}^nC_n a^0 b^n}$$

$(a+b)^n \leftarrow \text{Power} = \text{index} = \text{Exponent}$
 $n \in W = \text{whole No.}$

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, \dots, {}^nC_{n-1}, {}^nC_n = \text{Binomial Coefficients.}$

$\downarrow \quad \downarrow \quad \quad \quad \downarrow \quad \downarrow$
 $(1) \quad (n) \quad \quad \quad (n) \quad (1)$

Symmetry.

observations:

① $(a+b)^n = \sum_{r=0}^{r=n} \boxed{{}^nC_r a^{n-r} b^r}$

short form

$\Sigma \rightarrow \oplus$ (sigma)

Summation = $\oplus$

Upper limit $\rightarrow 10$
 Lower limit $\rightarrow r=5$

$$\sum_{r=5}^{10} (2^r) = 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10}$$

observations

② No. of terms in the expansion of $(a+b)^n = n+1$
= one more than index (n)

③ In Subsequent terms,

* power of $a = \downarrow$ decreases.
power of $b = \uparrow$ increases.

④ In each term, Sum of powers of a & b = n.
 $(a+b)^n$

Some Special Expansion.

$$(a+b)^n = {}^nC_0 \cdot \underline{a^n} \cdot \underline{b^0} + {}^nC_1 \cdot a^{n-1} \cdot \underline{b^1} + {}^nC_2 \cdot a^{n-2} \cdot \underline{b^2} + \dots + {}^nC_n \cdot \underline{a^0} \cdot \underline{b^n}$$

① Replace $\underline{a \rightarrow a}$, $\underline{b \rightarrow -b}$ ✓

$$(a-b)^n = \underline{{}^nC_0 \cdot a^n} - \underline{{}^nC_1 \cdot a^{n-1} \cdot b} + \underline{{}^nC_2 \cdot a^{n-2} \cdot b^2} - \dots + (-1)^n \cdot {}^nC_n \cdot \underline{a^0 \cdot b^n}$$

② Replace $\underline{a \rightarrow 1}$, $\underline{b \rightarrow x}$

$$(1+x)^n = \cancel{{}^nC_0 \cdot 1^n} + {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$$

③ Replace $\underline{a \rightarrow 1}$, $\underline{b \rightarrow -x}$

$$(1-x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - \dots + {}^nC_n (-x)^n$$

④ Replace $\underline{a \rightarrow 1}$, $\underline{b \rightarrow 1}$

$$\boxed{2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n} \quad \star$$

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Class - 11 - Maths Chapter - 8 Binomial Theorem

Revision: $(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_{n-1} a^1 b^{n-1} + {}^nC_n a^0 b^n$

$n =$ whole no.

$n =$ index

Binomial Coefficients $= {}^nC_r = \frac{n!}{(n-r)! r!}$ (${}^nC_r = {}^nC_{n-r}$)

Exercise 8.1

Q.1 $(1-2x)^5$ ~~$(a+b)^5$~~

By Binomial theorem:

$$(1-2x)^5 = {}^5C_0 (1)^5 (-2x)^0 + {}^5C_1 (1)^4 (-2x)^1 + {}^5C_2 (1)^3 (-2x)^2 + {}^5C_3 (1)^2 (-2x)^3 + {}^5C_4 (1)^1 (-2x)^4 + {}^5C_5 (1)^0 (-2x)^5$$

$\begin{matrix} \swarrow \text{odd} \\ \downarrow \text{odd} \end{matrix}$

$$= [1 \cdot 1 \cdot 1] - [5 \cdot 1 \cdot 2x] + [10 \cdot 1 \cdot 4x^2] - [10 \cdot 1 \cdot 8x^3] + [5 \cdot 1 \cdot 16x^4] - [1 \cdot 1 \cdot 32x^5]$$

$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

Even
 $(-1)^{\text{Even}} = +1$
Odd
 $(-1)^{\text{Odd}} = -1$

${}^5C_0 = 1 = {}^5C_5$
 ${}^5C_1 = 5 = {}^5C_4$
 ${}^5C_2 = 10 = {}^5C_3$

Q.2

$$\left(\frac{2}{x} - \frac{x}{2}\right)^{5 \rightarrow n}$$

a By Binomial Theorem:

$$= {}^5C_0 \left(\frac{2}{x}\right)^5 \left(-\frac{x}{2}\right)^0 + {}^5C_1 \left(\frac{2}{x}\right)^4 \left(-\frac{x}{2}\right)^1 + {}^5C_2 \left(\frac{2}{x}\right)^3 \left(-\frac{x}{2}\right)^2 + {}^5C_3 \left(\frac{2}{x}\right)^2 \left(-\frac{x}{2}\right)^3 \\ + {}^5C_4 \left(\frac{2}{x}\right)^1 \left(-\frac{x}{2}\right)^4 + {}^5C_5 \left(\frac{2}{x}\right)^0 \left(-\frac{x}{2}\right)^5$$

$$= 1 \cdot \frac{32}{x^5} \cdot 1 - 5 \cdot \frac{16}{x^4} \cdot \frac{x}{2} + 10 \cdot \frac{8}{x^3} \cdot \frac{x^2}{4} - 10 \cdot \frac{4}{x^2} \cdot \frac{x^3}{8} \\ + 5 \cdot \frac{2}{x} \cdot \frac{x^4}{16} - 1 \cdot 1 \cdot \frac{x^5}{32}$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^3}{8} - \frac{x^5}{32} \quad \checkmark$$

Q.3 $(2x-3)^6 \rightarrow n=6$
 $\downarrow \quad \searrow$
 $a=2x \quad b=-3$

$$6C_0 = 1 = 6C_6$$

$$6C_1 = 6 = 6C_5$$

$$6C_2 = 6C_4 = \frac{6!}{4!2!} = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1} = 15$$

$$= {}^6C_0 \cdot (2x)^6 \cdot (-3)^0 + {}^6C_1 \cdot (2x)^5 \cdot (-3)^1 + {}^6C_2 \cdot (2x)^4 \cdot (-3)^2 + {}^6C_3 \cdot (2x)^3 \cdot (-3)^3$$

$$+ {}^6C_4 \cdot (2x)^2 \cdot (-3)^4 + {}^6C_5 \cdot (2x)^1 \cdot (-3)^5 + {}^6C_6 \cdot (2x)^0 \cdot (-3)^6$$

$$6C_3 = \frac{6!}{3!3!}$$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3 \cdot 2} = 20$$

$$= 1 \cdot 64 \cdot x^6 \cdot 1 + 6 \cdot 32 \cdot x^5 \cdot (-3) + (15 \cdot 16 \cdot x^4 \cdot 9) + (20 \cdot 8 \cdot x^3 \cdot (-27))$$

$$+ (15 \cdot 4 \cdot x^2 \cdot 81) + (6 \cdot 2x \cdot (-243)) + (1 \cdot 1 \cdot 729)$$

$$\begin{array}{r} 243 \\ \times 3 \\ \hline 729 \end{array}$$

$$= 64x^6 - 586x^5 + 2160x^4 - 4320x^3$$

$$+ 4860x^2 - 2916x + 729$$

$$\begin{array}{r} 729 \\ \times 4 \\ \hline 216 \\ 432 \end{array}$$

$$\begin{array}{r} 18 \\ \times 32 \\ \hline 36 \\ 54 \\ \hline 586 \end{array}$$

$$\begin{array}{r} 729 \\ \times 4 \\ \hline 2916 \end{array}$$

$$\begin{array}{r} 81 \\ \times 6 \\ \hline 486 \end{array}$$

$$\begin{array}{r} 240 \\ \times 9 \\ \hline 2160 \end{array}$$

(Q. 4) $\left(\frac{x}{3} + \frac{1}{x}\right)^5 \rightarrow n=5$

$\swarrow \quad \searrow$
 $a = \frac{x}{3} \quad b = \frac{1}{x}$

By Binomial Theorem:

$$= {}^5C_0 \cdot \left(\frac{x}{3}\right)^5 \cdot \left(\frac{1}{x}\right)^0 + {}^5C_1 \cdot \left(\frac{x}{3}\right)^4 \cdot \left(\frac{1}{x}\right)^1 + {}^5C_2 \cdot \left(\frac{x}{3}\right)^3 \cdot \left(\frac{1}{x}\right)^2 + {}^5C_3 \cdot \left(\frac{x}{3}\right)^2 \cdot \left(\frac{1}{x}\right)^3$$

$$+ {}^5C_4 \cdot \left(\frac{x}{3}\right)^1 \cdot \left(\frac{1}{x}\right)^4 + {}^5C_5 \cdot \left(\frac{x}{3}\right)^0 \cdot \left(\frac{1}{x}\right)^5$$

$$= 1 \cdot \frac{x^5}{243} \cdot 1 + 5 \cdot \frac{x^4}{81} \cdot \frac{1}{x} + 10 \cdot \frac{x^3}{27} \cdot \frac{1}{x^2} + 10 \cdot \frac{x^2}{9} \cdot \frac{1}{x^3}$$

$$+ 10x \cdot \frac{x}{3} \cdot \frac{1}{x^4} + 1 \cdot 1 \cdot \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{10}{3x^3} + \frac{1}{x^5}$$

Q.5 $\left(x + \frac{1}{x}\right)^6 \rightarrow n=6$

$\swarrow$ $\searrow$
 a b

By Binomial Theorem:

$$\begin{aligned}
 &= {}^6C_0 \cdot \underbrace{\left(x\right)^6}_{\uparrow} \cdot \underbrace{\left(\frac{1}{x}\right)^0}_{\uparrow} + {}^6C_1 \cdot \left(x\right)^5 \cdot \left(\frac{1}{x}\right)^1 + {}^6C_2 \cdot \left(x\right)^4 \cdot \left(\frac{1}{x}\right)^2 + {}^6C_3 \cdot \cancel{\left(x\right)^3} \cdot \cancel{\left(\frac{1}{x}\right)^3} \\
 &\quad + {}^6C_4 \cdot \left(x\right)^2 \cdot \left(\frac{1}{x}\right)^4 + {}^6C_5 \cdot \left(x\right)^1 \cdot \left(\frac{1}{x}\right)^5 + {}^6C_6 \cdot \left(x\right)^0 \cdot \left(\frac{1}{x}\right)^6
 \end{aligned}$$

$$= 1 \cdot x^6 \cdot 1 + 6 \cdot x^4 + 15 \cdot x^2 + 20 \cdot 1 + \frac{15}{x^2} + 6 \cdot \frac{1}{x^4} + 1 \cdot 1 \cdot \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

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Class-11-Maths chapter-8 Binomial Theorem

$$\boxed{Q.6} \quad (96)^3 = \underset{\times}{(95+1)^3} = \underset{\checkmark}{(100-4)^3}$$

$$(a \pm b)^n = {}^nC_0 \cdot a^n \cdot b^0 \pm {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 \pm \dots + {}^nC_n \cdot a^0 \cdot b^n$$

$$\begin{aligned} (96)^3 &= (100-4)^3 = {}^3C_0 \cdot 100^3 \cdot (-4)^0 + {}^3C_1 \cdot 100^2 \cdot (-4)^1 + {}^3C_2 \cdot 100^1 \cdot (-4)^2 + {}^3C_3 \cdot 100^0 \cdot (-4)^3 \\ &\quad \begin{array}{ccccccc} & \uparrow & \uparrow & & \downarrow & & \downarrow \\ & a=100 & b=-4 & & 1 & & 16 \end{array} \\ &\quad \begin{array}{ccccccc} & & & & \downarrow & & \downarrow \\ & & & & 3 & & 16 \end{array} \\ &\quad \begin{array}{ccccccc} & & & & \downarrow & & \downarrow \\ & & & & 1 & & 1 \end{array} \\ &\quad \begin{array}{ccccccc} & & & & \downarrow & & \downarrow \\ & & & & 1 & & -64 \end{array} \\ &= 1 \cdot \underline{1000000} - 120000 + \underline{4800} - 64 \end{aligned}$$

$$(96)^3 = 884736$$

$$\begin{array}{r} + 1004800 \\ - 120064 \\ \hline \underline{884736} \end{array}$$

Q.7 $(102)^5 = (100+2)^5$

Binomial Theorem :

$$\begin{aligned} {}^5C_2 &= {}^5C_3 = 10 \\ {}^5C_0 &= 1 = {}^5C_5 \\ {}^5C_4 &= {}^5C_1 = 5 \end{aligned}$$

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

$$\begin{aligned} (100+2)^5 &= {}^5C_0 \cdot 100^5 \cdot 2^0 + {}^5C_1 \cdot 100^4 \cdot 2^1 + {}^5C_2 \cdot 100^3 \cdot 2^2 + {}^5C_3 \cdot 100^2 \cdot 2^3 + {}^5C_4 \cdot 100^1 \cdot 2^4 \\ &\quad + {}^5C_5 \cdot 100^0 \cdot 2^5 \end{aligned}$$

$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$

$$= 11040808032$$

Q.8. $(101)^4 = (100+1)^4$

$$= {}^4C_0 \cdot 100^4 \cdot 1^0 + {}^4C_1 \cdot 100^3 \cdot 1^1 + {}^4C_2 \cdot 100^2 \cdot 1^2 + {}^4C_3 \cdot 100^1 \cdot 1^3 + {}^4C_4 \cdot 100^0 \cdot 1^4$$

$$= 100000000 + 4000000 + 60000 + 400 + 1$$

$$= 104060401 \checkmark$$

$$\begin{aligned}
 \textcircled{9} \quad (99)^5 &= (100 - 1)^5 = \left[\underset{a}{100} + \underset{b}{(-1)} \right]^5 \\
 &= {}^5C_0 \cdot (100)^5 \cdot (-1)^0 + {}^5C_1 \cdot 100^4 \cdot (-1)^1 + {}^5C_2 \cdot 100^3 \cdot \overbrace{(-1)^2}^{+1} + {}^5C_3 \cdot 100^2 \cdot (-1)^3 \\
 &\quad + {}^5C_4 \cdot (100)^1 \cdot (-1)^4 + {}^5C_5 \cdot 100^0 \cdot (-1)^5
 \end{aligned}$$

$$\begin{aligned}
 &= \textcircled{10000000000} - \underline{500000000} + \textcircled{10000000} - 100000 \\
 &\quad + \textcircled{500} - 1
 \end{aligned}$$

$$= 9509900499$$

$$\begin{array}{r}
 \underline{10010000500} \\
 - \quad \underline{500100001} \\
 \hline
 9509900499
 \end{array}$$

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Class - 11 - Maths Exercise (8.1)

Binomial Theorem:

$$(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 + \dots + {}^nC_n \cdot a^0 \cdot b^n$$

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

[Q.10] Find larger Number.

$$(1.1)^{10000}$$

or 1000

$$(1.1)^{10000} = (1 + 0.1)^{10000}$$

$$= {}^{10000}C_0 \cdot 1^{10000} \cdot (0.1)^0 + {}^{10000}C_1 \cdot 1^{9999} \cdot (0.1)^1 + \text{other positive Terms.}$$

Diagram showing the expansion of the binomial theorem with arrows pointing from the terms to their components:

- ${}^{10000}C_0$ points to 1
- 1^{10000} points to 1
- $(0.1)^0$ points to 1
- ${}^{10000}C_1$ points to 10000
- 1^{9999} points to 1
- $(0.1)^1$ points to 0.1

The result of the first two terms is circled and labeled 1000.

$$= 1 + 1000 + \text{other positive Terms.}$$

$$= \text{1000} + \text{other positive Terms} > 1000$$

$$(1.1)^{10000} > 1000$$

Q.11

$$(a+b)^4 - (a-b)^4 = ?$$

$$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = ?$$

$$(a+b)^4 = {}^4C_0 \cdot a^4 \cdot b^0 + {}^4C_1 \cdot a^3 \cdot b^1 + {}^4C_2 \cdot a^2 \cdot b^2 + {}^4C_3 \cdot a^1 \cdot b^3 + {}^4C_4 \cdot a^0 \cdot b^4$$

$$(a-b)^4 = {}^4C_0 \cdot a^4 \cdot b^0 - {}^4C_1 \cdot a^3 \cdot b^1 + {}^4C_2 \cdot a^2 \cdot b^2 - {}^4C_3 \cdot a^1 \cdot b^3 + {}^4C_4 \cdot a^0 \cdot b^4$$

$$(a+b)^4 - (a-b)^4 = 2 ({}^4C_1 \cdot a^3 \cdot b^1 + {}^4C_3 \cdot a^1 \cdot b^3)$$

$$\begin{aligned} {}^4C_1 &= 4 \\ {}^4C_3 &= 4 \end{aligned}$$

$$(a+b)^4 - (a-b)^4 = 2(4a^3b + 4a \cdot b^3)$$

$$(a+b)^4 - (a-b)^4 = 8ab(a^2 + b^2)$$

$$\text{put } a = \sqrt{3} / b = \sqrt{2}$$

$$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8 \cdot \sqrt{3} \cdot \sqrt{2} [(\sqrt{3})^2 + (\sqrt{2})^2]$$

$$= 8 \cdot \sqrt{6} \cdot (3 + 2)$$

$$= 40 \cdot \sqrt{6}$$

Q.12 $(x+1)^6 + (x-1)^6 = ?$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$$(x+1)^6 = {}^6C_0 \cdot x^6 \cdot 1^0 + {}^6C_1 \cdot x^5 \cdot 1^1 + {}^6C_2 \cdot x^4 \cdot 1^2 + {}^6C_3 \cdot x^3 \cdot 1^3 + {}^6C_4 \cdot x^2 \cdot 1^4 + {}^6C_5 \cdot x^1 \cdot 1^5 + {}^6C_6 \cdot x^0 \cdot 1^6$$

$$(x-1)^6 = {}^6C_0 \cdot x^6 \cdot 1^0 - {}^6C_1 \cdot x^5 \cdot 1^1 + {}^6C_2 \cdot x^4 \cdot 1^2 - {}^6C_3 \cdot x^3 \cdot 1^3 + {}^6C_4 \cdot x^2 \cdot 1^4 - {}^6C_5 \cdot x^1 \cdot 1^5 + {}^6C_6 \cdot x^0 \cdot 1^6$$

+

$$(x+1)^6 + (x-1)^6 = 2 \left(\underbrace{{}^6C_0}_{1} \cdot x^6 + \underbrace{{}^6C_2}_{15} \cdot x^4 + \underbrace{{}^6C_4}_{15} \cdot x^2 + \underbrace{{}^6C_6}_{1} \right)$$

$${}^6C_2 = \frac{6!}{4!2!} = 15$$

$$= \frac{3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = 1$$

$$(x+1)^6 + (x-1)^6 = 2(x^6 + 15x^4 + 15x^2 + 1)$$

Put $x = \sqrt{2}$

$$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2 \left((\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1 \right)$$

$$= 2(2^3 + 15 \cdot 2^2 + 15 \cdot 2 + 1)$$

$$= 2(8 + 60 + 30 + 1)$$

$$= 2 \times 99 = 198$$

$$\left((2)^{\frac{1}{2}} \right)^6 = 2^3$$

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Exercise 8.1 class - 11 - Maths.

$$(a+b)^n = \underline{{}^nC_0 \cdot a^n \cdot b^0} + \underline{{}^nC_1 \cdot a^{n-1} \cdot b^1} + \underline{{}^nC_2 \cdot a^{n-2} \cdot b^2} + \dots + \underline{{}^nC_n \cdot a^0 \cdot b^n}$$
$$= \sum_{r=0}^{r=n} \boxed{{}^nC_r \cdot a^{n-r} \cdot b^r} \quad \text{variable} = r$$

Division Algorithm $\Rightarrow$ Dividend = Divisor $\times$ Quotient
+ Remainder.

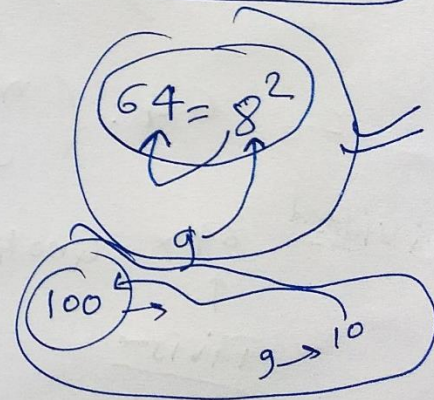
Q.13 Dividend = $9^{n+1} - 8n - 9$, Divisor = 64, Prove $\star$
Remainder = 0

$$\text{Dividend} = \underline{9^{n+1} - 8n - 9} = 64 \times \boxed{} + \boxed{}^0$$

~~$= 8$~~

$$= (1+8)^{n+1} - 8n - 9$$

Expand $\rightarrow$ By Binomial Theorem.



$$\text{Dividend} = 9^{n+1} - 8n - 9$$

$$= (1+8)^{n+1} - 8n - 9$$

$${}^{n+1}C_0 = 1$$

$${}^{n+1}C_1 = n+1$$

$$64 = 8^2$$

$$64 \times 8$$

$$8^3 = 512$$

$$= {}^{n+1}C_0 \cdot 1^{n+1} \cdot 8^0 + {}^{n+1}C_1 \cdot 1^n \cdot 8^1 + {}^{n+1}C_2 \cdot 1^{n-1} \cdot 8^2 + {}^{n+1}C_3 \cdot 1^{n-2} \cdot 8^3 + \dots + {}^{n+1}C_{n+1} \cdot 1^0 \cdot 8^{n+1} - 8n - 9$$

$$= (1 \cdot 1 \cdot 1) + (n+1) \cdot 1 \cdot 8 + \underbrace{{}^{n+1}C_2 \cdot 1 \cdot (64) + {}^{n+1}C_3 \cdot 1 \cdot (64) \cdot 8 + \dots + {}^{n+1}C_{n+1} \cdot 1 \cdot (64) \cdot 8^{n-1}}_{64 \text{ Common}}$$

$$= \cancel{1} + \cancel{8n} + \cancel{8} + 64 \cdot \underbrace{({}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + {}^{n+1}C_{n+1} \cdot 8^{n-1})}_{\text{Integer}}$$

$$= 64 \cdot ({}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + {}^{n+1}C_{n+1} \cdot 8^{n-1}) + 0$$

Dividend

64 x Quotient
↑
Divisor

0
↑
Remainder

∴ $9^{n+1} - 8n - 9$ is divisible
by 64 (∵ remainder = 0)

Class-11-Maths. Binomial Theorem.

Basics of Exercise 8.2

$$(a+b)^n = \underbrace{{}^nC_0 \cdot a^n \cdot b^0}_{1^{st}} + \underbrace{{}^nC_1 \cdot a^{n-1} \cdot b^1}_{2^{nd}} + \underbrace{{}^nC_2 \cdot a^{n-2} \cdot b^2}_{3^{rd}} + \dots + \underbrace{{}^nC_n \cdot a^0 \cdot b^n}_{(n+1)^{th}}$$

$a^0 = a^{n-n}$
↑

Total no. of Terms = n+1

$$r^{th} \text{ Term} = {}^nC_{r-1} \cdot a^{n-(r-1)} \cdot b^{r-1} = \underbrace{{}^nC_{r-1} \cdot a^{n-r+1} \cdot b^{r-1}}_{\text{Complicated expression.}}$$

$$\text{General Term} = T_{r+1} = \boxed{{}^nC_r \cdot a^{n-r} \cdot b^r}$$

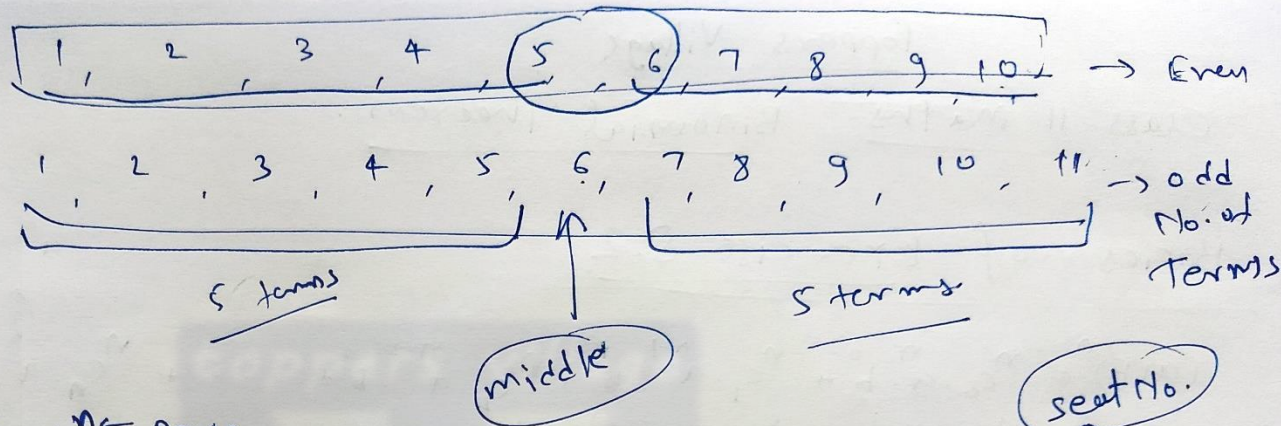
$$T_7 = T_{6+1} = {}^nC_6 \cdot \underbrace{a^{n-6} \cdot b^6}_{\text{coeff.}}$$

use → coeff. = ?
→ particular Term =

Middle Term:

Tough

easy



Middle Term in $(a+b)^n$ ← power

	No. of Terms = $(n+1)$	No. of Middle terms	which terms are the <u>Rank</u> ^{seral No.}
$n = \text{Even}$	$n+1 = \text{odd}$	1	$\left(\frac{(n+1)+1}{2}\right)^{\text{th}}$ term = $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term
$n = \text{odd}$	$n+1 = \text{even}$	2	$\left(\frac{n+1}{2}\right)^{\text{th}}$ term, $\left(\frac{n+1}{2} + 1\right)^{\text{th}}$ term

e.g. middle term in $(a+b)^{11}$ $n=11$ No. of middle terms = 2

middle Term $\Rightarrow T_{\frac{11+1}{2}} = T_6 = {}^{11}C_5 \cdot a^{11-5} \cdot b^5$

middle term $\Rightarrow T_{\left(\frac{11+1}{2} + 1\right)} = T_7 = T_{6+1} = {}^{11}C_6 \cdot a^{11-6} \cdot b^6$

Note:

Middle Term in the expansion

of $\left(x + \frac{1}{x}\right)^{2n}$, $x \neq 0$

$$a = x, b = \frac{1}{x}$$

Power = $2n$ = Even.

$$\text{middle Term} = \text{the } \left(\frac{2n}{2} + 1\right)^{\text{th}} \text{ term} = (n+1)^{\text{th}} \text{ term} = T_{n+1}$$

$$\rightarrow T_{n+1} = {}^{2n}C_n \cdot a^{2n-n} \cdot b^n = {}^{2n}C_n \cdot a^n \cdot b^n$$

$$\text{middle term in } \left(x + \frac{1}{x}\right)^{2n} = T_{n+1} = {}^{2n}C_n \cdot \cancel{\left(x\right)^n} \cdot \cancel{\left(\frac{1}{x}\right)^n}$$

$$= {}^{2n}C_n \cdot x^0$$

$$= {}^{2n}C_n = \text{term independent of 'x'}$$

power of variable $x = 0$

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Class-II-Maths Exercise-8.2

General Term in the expansion of $(a+b)^n$

$$T_{r+1} = (r+1)^{th} \text{ Term} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

$$r=0, T_1 = {}^nC_0 \cdot a^n \cdot b^0$$

$$r=1, T_2 = {}^nC_1 \cdot a^{n-1} \cdot b^1$$

use → Coefficient ✓
→ General Term ✓
→ Particular Term ✓

Exercise-8.2

[Q.1] coefficient of x^5 in $(x+3)^8 \rightarrow n=8$

(I)
General Term

General Term = $T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

$$T_{r+1} = {}^nC_r \cdot \underbrace{(x^{8-r})}_{\text{variable}} \cdot 3^r$$

$$T_{r+1} = \underbrace{{}^nC_r \cdot 3^r} \cdot x^{8-r}$$

$$T_{r+1} = {}^nC_r \cdot 3^r \cdot x^{8-r}$$

For x^5 $5 = 8-r$

Comparing x^{8-r} & x^5

$$\Rightarrow r = 8-5$$

$$r=3$$

Put $r=3$ in General Term

$$T_{3+1} = {}^nC_3 \cdot 3^3 \cdot x^{8-3}$$

$$T_4 = {}^nC_3 \cdot 3^3 \cdot x^5$$

$$\text{Coeff. of } x^5 = {}^nC_3 \cdot 3^3$$

$$= \frac{8!}{5!3!} \times 27$$

$$= \underline{\hspace{2cm}}$$

Q.2 coefficient of $a^5 b^7$ in $(a-2b)^{12}$

$\downarrow \quad \uparrow$
 $(A+B)^n$

General Term in the expansion
of $(a-2b)^{12} =$

$$T_{r+1} = {}^n C_r \cdot A^{n-r} \cdot B^r$$

$$T_{r+1} = {}^{12} C_r \cdot \underbrace{a^{12-r}}_{\uparrow} \cdot \underbrace{(-2b)^r}_{\uparrow}$$

$$\boxed{T_{r+1} = \underbrace{{}^{12} C_r \cdot (-2)^r}_{\text{coeff.}} \cdot \underbrace{a^{12-r} b^r}_{\text{Variable}}}_{\text{General Term}}$$

~~for~~ for $a^5 b^7$

$$a^{(12-r)} \cdot b^{\boxed{r}} = a^{\boxed{5}} \cdot b^{\boxed{7}}$$

(a)

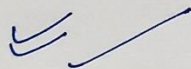
$$12-r = 5$$

$$12-5 = 7$$

$$\boxed{r=7}$$

(b)

$$r = 7$$



Put $r=7$ in General Term:

$$T_{r+1} = T_{7+1} = {}^{12} C_7 \cdot (-2)^7 \cdot a^{12-7} \cdot b^7$$

$$T_8 = {}^{12} C_7 \cdot \underbrace{(-2)^7}_{\uparrow} \cdot \underline{a^5 b^7}$$

$$\text{Coeff. of } a^5 b^7 = {}^{12} C_7 \times (-128)$$

$$\underline{\text{Coeff. of } a^1 b^2 = 0}$$

$$\text{Ox } a^1 b^2$$

③ General Term in the expansion of $(x^2 - y)^6 \rightarrow n$

General

$$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

$$T_{r+1} = {}^6C_r \cdot (x^2)^{6-r} \cdot (-y)^r$$

$$-y = -1xy$$

$$T_{r+1} = {}^6C_r \cdot x^{12-2r} \cdot [(-1) \cdot y]^r$$

$$T_{r+1} = {}^6C_r \cdot (-1)^r \cdot x^{12-2r} \cdot y^r$$

$$\cancel{x^{24-2r}} \cdot \cancel{y^r} \cdot \cancel{y^r}$$

④ General Term in the expansion of $(x^2 - yx)^{12} \rightarrow n$

$$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

General Term

$$T_{r+1} = {}^{12}C_r \cdot (x^2)^{12-r} \cdot (-yx)^r$$

$$T_{r+1} = {}^{12}C_r \cdot x^{24-2r} \cdot (-1)^r \cdot y^r \cdot x^r$$

$$T_{r+1} = (-1)^r \cdot {}^{12}C_r \cdot x^{24-r} \cdot y^r$$

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Class-11-Maths Exercise-8.2

General Term $T_{r+1} = \frac{nCr \cdot a^{n-r} \cdot b^r}{\text{in } (a+b)^n}$

Q.5 4th Term in $(x-2y)^{12} \rightarrow n=12$

$\downarrow$ $\downarrow$
 a b

$$T_4 = T_{3+1} = {}^{12}C_3 \cdot (x)^{12-3} \cdot (-2y)^3$$

$\begin{matrix} 4=r+1 \\ r=3 \end{matrix}$ ✓ $T_4 = {}^{12}C_3 \cdot x^9 \cdot (-2)^3 \cdot y^3$

$$T_4 = \left(\frac{12!}{9! 3!} \right) \times (-8) \cdot x^9 \cdot y^3$$

$$T_4 = \frac{2 \times 12 \times 11 \times 10 \times 9!}{9! \times 3 \times 2 \times 1} \times (-8) \cdot x^9 \cdot y^3$$

$$T_4 = -1760 x^9 y^3$$

Q.6

13th term in $\left(9x - \frac{1}{3\sqrt{x}} \right)^{18}$, $x \neq 0$

$\uparrow$ $\uparrow$
 a b

$$T_{13} = T_{12+1} = {}^{18}C_{12} \cdot (9x)^{18-12} \cdot \left(\frac{-1}{3\sqrt{x}} \right)^{12}$$

$\swarrow$
 $r=12$

$$T_{13} = {}^{18}C_{12} \cdot 9^6 \cdot x^6 \cdot \frac{1}{3^{12} \cdot (\sqrt{x})^{12}}$$

$$T_{13} = {}^{18}C_{12} \cdot \frac{9^{12}}{3^{12}} \cdot \frac{x^6}{x^6} \cdot \frac{1}{1} \quad \left((\sqrt{x})^{12} = x^6 \right)$$

$$T_{13} = {}^{18}C_{12}$$

$$T_{13} = \frac{18!}{6! \times 12!} \quad \checkmark$$

Q.7 Middle Term in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$

Index = 7 = odd
(Power)
(Exponent)

No. of terms = 8 = Even

$$\underline{T_1 + T_2 + T_3 + (T_4 + T_5) + T_6 + T_7 + T_8}$$

Middle Terms = T_4, T_5

Middle Terms $\rightarrow T_{\frac{n+1}{2}}, T_{\frac{n+1}{2} + 1}$
 $\text{index } n = \text{odd}$
 $n = 7$
 Middle terms $\rightarrow T_{\frac{7+1}{2}}, T_{\frac{7+1}{2} + 1}$
 $= T_4, T_5$

$$T_4 = T_{3+1} = {}^7C_3 \cdot (3)^{7-3} \cdot \left(-\frac{x^3}{6}\right)^3$$

$$= - {}^7C_3 \cdot 3^4 \cdot \frac{x^9}{6^3} \quad (6)^3 = (2 \times 3)^3$$

$$T_4 = - {}^7C_3 \cdot \cancel{3^4} \cdot \frac{x^9}{2^3 \cdot \cancel{3^3}}$$

$$T_4 = - \frac{{}^7C_3}{2^3} \cdot 3x^9$$

$$\textcircled{2^3 = 8}$$

Q.8 Middle Term in $\left(\frac{x}{3} + \frac{y}{x}\right)^{10}$

$$T_5 = T_{4+1} = {}^7C_4 \cdot 3^{7-4} \cdot \left(\frac{-x^3}{6}\right)^4$$

$$= + {}^7C_4 \cdot 3^3 \cdot \frac{x^{12}}{6^4}$$

$$= {}^7C_4 \cdot \cancel{3^3} \cdot \frac{x^{12}}{2^4 \cdot \cancel{3^4}} \cdot \frac{1}{3}$$

$$= \frac{{}^7C_4 \cdot x^{12}}{2^4 \cdot 3}$$

$$\textcircled{2^4 = 16}$$

$${}^7C_4 = {}^7C_3 = \frac{7!}{3! 4!}$$

$$\textcircled{nCr = nC_{n-r}}$$

Q.8 middle Term in $\left(\frac{x}{3} + 9y\right)^{10}$

$n = 10 = \text{index} = \text{Even}$.

No. of Terms = 11 = odd

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad \boxed{T_6} \quad T_7 \quad T_8 \quad T_9 \quad T_{10} \quad T_{11}$
middle term.

$$\text{middle term} = T_{\frac{n}{2}+1} = T_{\frac{10}{2}+1} = T_{5+1} = T_6$$

$n = \text{Even}$

$$T_6 = T_{5+1} = {}^{10}C_5 \cdot \left(\frac{x}{3}\right)^{10-5} \cdot (9y)^5$$

$$T_6 = {}^{10}C_5 \cdot \left(\frac{x}{3}\right)^5 \cdot 9^5 \cdot y^5$$

$$T_6 = {}^{10}C_5 \cdot \frac{x^5}{3^5} \cdot \frac{3^5 \cdot 10}{2^5} \cdot y^5$$

$$T_6 = {}^{10}C_5 \cdot 3^5 \cdot x^5 \cdot y^5$$

$$3^5 = 243$$

$${}^{10}C_5 = \frac{10!}{5!5!}$$

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Class-11-maths. Exercise 8.2

General Term in $(A+B)^n$

$$(r+1)^{th} \text{ term} = T_{r+1} = {}^nC_r \cdot A^{n-r} \cdot B^r$$

Q.9 $(1+a)^{m+n}$

To Prove $\boxed{\text{Coeff. of } a^m = \text{Coeff. of } a^n}$

Proof: General Term of $(1+a)^{m+n}$

$$T_{r+1} = {}^{m+n}C_r \cdot (1)^{m+n-r} \cdot (a)^r$$

$$\boxed{T_{r+1} = {}^{m+n}C_r \cdot a^r} \leftarrow \text{General Term}$$

$$\text{coeff. of } a^0 = {}^{m+n}C_0$$

Similarly $\text{coeff. of } a^m = {}^{m+n}C_m \checkmark$

$$\text{coeff. of } a^n = {}^{m+n}C_n \checkmark$$

$$\text{coeff. of } a^m = {}^{m+n}C_m$$

Property

$${}^nC_r = {}^nC_{n-r}$$

$$= {}^{m+n}C_{(m+n)-(m)}$$

$$= {}^{m+n}C_n$$

$$= \text{Coeff. of } a^n$$

Q.10 (coeff. of $(r-1)^{th}$ term) : (coeff. of r^{th} Term) : (coeff. of $(r+1)^{th}$ Term) = 1:3:5

$(r+1)^{th}$ Term = $T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$
(General Term)

$\Rightarrow T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot 1^r = \underbrace{{}^nC_r}_{\text{coeff.}} \cdot \underbrace{x^{n-r}}_{\text{variable}}$

~~r^{th} Term = $T_r = {}^nC_{r-1}$~~

According to Question:

$\underbrace{{}^nC_{r-2} : {}^nC_{r-1} : {}^nC_r}_{1:3:5}$

to be continued.

in $(x+1)^n$
a b

coeff. of $(r+1)^{th}$ Term
= nC_r ✓

coeff. of r^{th} term
= ${}^nC_{r-1}$ ✓

coeff. of $(r-1)^{th}$ term
= ${}^nC_{r-2}$ ✓

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Class-11-Math Exercise-8.2

Q.11

To Prove

$$\text{coeff. of } x^n \text{ in } (1+x)^{2n} = 2 \times \text{coeff. of } x^n \text{ in } (1+x)^{2n-1}$$

$$\underline{\underline{2^n C_n = 2 \times 2^{n-1} C_n}}$$

$$\text{General Term} = T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$$

$$(a+b)^n$$

$$a=1, b=x$$

General Term of $(1+x)^{2n}$

$$T_{r+1} = {}^{2n}C_r \cdot (1)^{2n-r} \cdot x^r$$

$$T_{r+1} = {}^{2n}C_r \cdot x^r$$

$$\text{coeff. of } x^r = {}^{2n}C_r$$

$$\text{coeff. of } x^n = {}^{2n}C_n \quad \checkmark$$

General Term of $(1+x)^{2n-1}$

$$t_{r+1} = {}^{2n-1}C_r \cdot (1)^{2n-1-r} \cdot x^r$$

$$t_{r+1} = {}^{2n-1}C_r \cdot x^r$$

$$\text{coeff. of } x^r = {}^{2n-1}C_r$$

$$\text{coeff. of } x^n = {}^{2n-1}C_n$$

... to be continued

$$n_{C_{r-2}} : n_{C_{r-1}} = 1:3$$

$$\Rightarrow \frac{n_{C_{r-2}}}{n_{C_{r-1}}} = \frac{1}{3}$$

$$\Rightarrow \frac{\frac{n!}{(n-r+2)!(r+2)!}}{\frac{n!}{(n-r+1)!(r+1)!}} = \frac{1}{3}$$

$$(r+1)! = (r+1) \cdot (r)! \quad (r-1)! = (r-1) \cdot (r-2)!$$

$$(n-r+2)! = (n-r+2) \cdot (n-r+1)!$$

$$\Rightarrow \frac{\frac{1}{(n-r+2)}}{\frac{1}{(r+1)}} = \frac{1}{3}$$

$$\Rightarrow \frac{r+1}{n-r+2} = \frac{1}{3}$$

$$\Rightarrow 3r+3 = n-r+2$$

$$\Rightarrow \boxed{4r-5=n} \quad \text{--- (1)}$$

similarly,

$$n_{C_{r-1}} : n_{C_r} = 3:5$$

$$\Rightarrow \frac{n_{C_{r-1}}}{n_{C_r}} = \frac{3}{5}$$

$$\Rightarrow \frac{\frac{n!}{(n-r+1)!(r+1)!}}{\frac{n!}{(n-r)!(r)!}} = \frac{3}{5}$$

$$\Rightarrow \frac{\frac{1}{n-r+1}}{\frac{1}{r}} = \frac{3}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{5} \Rightarrow 5r = 3n - 3r + 3$$

$$\Rightarrow 8r - 3 = 3n \quad \text{--- (2)}$$

By eqn. (1) & (2)

$$\begin{array}{r} 8r - 3 = 3n \\ -8r + 10 = -2n \\ \hline 7 = n \end{array}$$

$$n=7 \checkmark$$

$$r=3 \checkmark$$

Now we have to prove that

$$\frac{2n}{n} C_n = 2 \cdot \frac{2n-1}{n} C_n$$

$$\frac{(2n)!}{n! (n!)}$$

$$\cancel{2n-1-n=n-1}$$

$$(n-1)! \times \underset{\uparrow}{n} = n!$$

$$RHS = 2 \cdot \frac{2n-1}{n} C_n$$

$$= 2 \times \frac{(2n-1)!}{(n-1)! (n!)} \times \frac{n}{n} C_n$$

$$= \frac{(2n) \cdot (2n-1)!}{n! (n!)}$$

$$= \frac{(2n)!}{n! n!}$$

$$= \frac{2n}{n} C_n = LHS.$$

Q.12 Find m

coeff. of x^2 in $(1+x)^m = 6$

$\downarrow$ $\downarrow$
 a b

$m \rightarrow m=n$

General Term $T_{r+1} = {}^m C_r \cdot a^{m-r} \cdot b^r$

$$T_{r+1} = {}^m C_r \cdot \underbrace{1^{m-r}}_1 \cdot x^r$$

$$T_{r+1} = {}^m C_r \cdot x^r$$

coeff. of $x^r = {}^m C_r$

coeff. of $x^2 = {}^m C_2 = 6$

$$\Rightarrow \frac{m!}{(m-2)! 2!} = 6$$

$$\Rightarrow \frac{m \times (m-1) \times \cancel{(m-2)!}}{\cancel{(m-2)!} \times 2 \times 1} = 6$$

$$\Rightarrow \frac{m^2 - m}{2} = 6$$

$$m^2 - m = 12$$

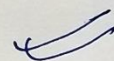
$$\Rightarrow m^2 - m - 12 = 0$$

$$\Rightarrow \underline{m^2 - 4m + 3m - 12 = 0}$$

$$\Rightarrow m(m-4) + 3(m-4) = 0$$

$$\Rightarrow (m-4)(m+3) = 0$$

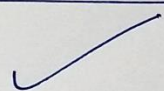
$$m = 4$$



$$m = -3$$



Positive value of $m = 4$



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Class - 11 - Maths Miscellaneous Exercise
on Chapter 8

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

$${}^nC_0 = 1$$

$${}^nC_1 = n$$

$${}^nC_2 = \frac{n!}{(n-2)! 2!} = \frac{n(n-1)}{2}$$

Q.1 In the expansion of $(a+b)^n = {}^nC_0 \cdot a^n \cdot b^0 + {}^nC_1 \cdot a^{n-1} \cdot b^1 + {}^nC_2 \cdot a^{n-2} \cdot b^2 + \dots$

First Term = $T_1 = 729 = a^n$ — (1)

Second Term = $T_2 = 7290 = n \cdot a^{n-1} \cdot b$ — (2)

Third Term = $T_3 = 30375 = \frac{n(n-1)}{2} \cdot a^{n-2} \cdot b^2$ — (3)

By eqⁿ (1), $\frac{729}{7290} = \frac{a^n}{n \cdot a^{n-1} \cdot b}$

$$\Rightarrow \frac{1}{10} = \frac{a}{nb}$$

$$\Rightarrow \boxed{nb = 10a} \text{ — (4)}$$

By eqⁿ (2), $\frac{7290}{30375} = \frac{n \cdot a^{n-1} \cdot b}{\frac{n(n-1)}{2} \cdot a^{n-2} \cdot b^2}$

$$\Rightarrow \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{2a}{(n-1)b}$$

$$\Rightarrow \frac{3}{25} = \frac{a}{(n-1)b}$$

$$\Rightarrow \boxed{3(n-1)b = 25a} \text{ — (5)}$$

$$\begin{array}{l} \text{By eqn (4)} \\ \text{eqn (5)} \end{array} \Rightarrow \frac{n}{3(n-1)} = \frac{2 \times 10 \times 9}{25 \times 5}$$

$$\Rightarrow \frac{n}{3(n-1)} = \frac{2}{5}$$

$$\Rightarrow 5n = 6n - 6$$

$$\Rightarrow 6 = 6n - 5n$$

$$\Rightarrow \boxed{n=6} \checkmark$$

$$\text{By eqn (1): } a^n = 729$$

$$\Rightarrow a^6 = 3^6$$

$$\Rightarrow \boxed{a=3}$$

$$a=3 \checkmark$$

$$n=6 \checkmark$$

$$\text{By eqn (2):}$$

$$7290 = n \cdot a^{n-1} \cdot b$$

$$\Rightarrow 7290 = 6 \cdot 3^5 \cdot b$$

$$\Rightarrow \cancel{3^6} \times \cancel{2} \times 5 = \cancel{2} \times \cancel{3} \times \cancel{3^5} \cdot b$$

$$\boxed{b=5} \checkmark$$

Q.2 $(3+ax)^9$ $\leftarrow n$ coeff. of x^2 = coeff. of x^3 — (3)
Given.

General Term:

$$T_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r \leftarrow (a+b)^n$$

$$T_{r+1} = {}^9C_r \cdot 3^{9-r} \cdot (ax)^r$$

$$T_{r+1} = {}^9C_r \cdot 3^{9-r} \cdot a^r \cdot \underbrace{(x^r)}_{\text{arrow}}$$

$\textcircled{9}$

$$\text{coeff. of } x^r = {}^9C_r \cdot 3^{9-r} \cdot a^r$$

$$\text{coeff. of } x^2 = {}^9C_2 \cdot 3^7 \cdot a^2 \text{ — (1)}$$

$$\text{coeff. of } x^3 = {}^9C_3 \cdot 3^6 \cdot a^3 \text{ — (2)}$$

By Question: ${}^9C_2 \cdot \cancel{3} \cdot \cancel{a^2} = {}^9C_3 \cdot \cancel{3} \cdot \textcircled{a}$

$$a = \frac{9}{7} \quad \checkmark$$

$$\Rightarrow \frac{{}^9C_2 \times 3}{{}^9C_3} = a = \frac{\frac{9!}{1!7!} \times 3}{\frac{9!}{6!3!3}} = \frac{\frac{3}{7} \times 3}{\frac{1}{3}} = \frac{9}{7}$$

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Class - 11 - Maths Misc. Exercise (8)

[Q.3] Coeff. of x^5 in the product $(1+2x)^6 \cdot (1-x)^7$

$$(1+2x)^6 = \left[{}^6C_0 \cdot (2x)^0 + {}^6C_1 \cdot (2x)^1 + {}^6C_2 \cdot (2x)^2 + {}^6C_3 \cdot (2x)^3 + {}^6C_4 \cdot (2x)^4 + {}^6C_5 \cdot (2x)^5 + {}^6C_6 \cdot (2x)^6 \right]$$

$$(1-x)^7 = \left[{}^7C_0 \cdot (-x)^0 + {}^7C_1 \cdot (-x)^1 + {}^7C_2 \cdot (-x)^2 + {}^7C_3 \cdot (-x)^3 + {}^7C_4 \cdot (-x)^4 + {}^7C_5 \cdot (-x)^5 + {}^7C_6 \cdot (-x)^6 + {}^7C_7 \cdot (-x)^7 \right]$$

x

$$(1+2x)^6 \times (1-x)^7 = [\quad] \times [\quad]$$

Coeff. of x^5 in $(1+2x)^6 \cdot (1-x)^7 =$ coeff. of $(x^5)^*$ in $[\quad] \times [\quad]$

$$\begin{aligned} \hookrightarrow &= \left({}^6C_0 \cdot 2^0 \right) \left(-{}^7C_5 \right) + \left({}^6C_1 \cdot 2^1 \right) \left({}^7C_4 \right) + \left({}^6C_2 \cdot 2^2 \right) \left(-{}^7C_3 \right) + \left({}^6C_3 \cdot 2^3 \right) \left({}^7C_2 \right) \\ &+ \left({}^6C_4 \cdot 2^4 \right) \left(-{}^7C_1 \right) + \left({}^6C_5 \cdot 2^5 \right) \left({}^7C_0 \right) = 171 \text{ Answer} \end{aligned}$$

$nCr = \frac{n!}{(n-r)!r!}$	${}^6C_0 = 1, {}^6C_1 = 6, {}^6C_2 = 15, {}^6C_3 = 20$ $\downarrow \quad \quad \downarrow \quad \quad \downarrow$ ${}^6C_5 \quad \quad {}^6C_4 \quad \quad {}^6C_3$	${}^7C_0 = 1, {}^7C_1 = 7$	${}^7C_2 = 21 = {}^7C_5, {}^7C_3 = 35 = {}^7C_4$	$nCr = nC_{n-r}$
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Q.4 Prove that $(a-b)$ is a factor of $a^n - b^n$

2-methods

Binomial

PMI
ch-4

$$a^n - b^n = (a-b+b)^n - b^n$$

$$= \left[\underbrace{(a-b)}_{\text{A}} + \underbrace{(b)}_{\text{B}} \right]^n - b^n$$

$${}^nC_n = 1 = {}^nC_0$$

$$= \left[{}^nC_0 \cdot (a-b)^n \cdot b^0 + {}^nC_1 \cdot (a-b)^{n-1} \cdot b^1 + {}^nC_2 \cdot (a-b)^{n-2} \cdot b^2 + \dots + {}^nC_{n-1} \cdot (a-b)^1 \cdot b^{n-1} + {}^nC_n \cdot (a-b)^0 \cdot b^n \right] - b^n$$

$$= \left[(a-b)^n + {}^nC_1 \cdot (a-b)^{n-1} \cdot b + \dots + {}^nC_{n-1} \cdot (a-b)^1 \cdot b^{n-1} + \cancel{b^n} \right] - \cancel{b^n}$$

$$= (a-b) \left\{ (a-b)^{n-1} + {}^nC_1 \cdot (a-b)^{n-2} \cdot b + \dots + {}^nC_{n-1} \cdot b^{n-1} \right\}$$

$$\underbrace{a^n - b^n}_{\text{factor}} = (a-b) \cdot \underbrace{\left\{ \text{Integer} \right\}}_{\substack{a, b, n \rightarrow \text{integers} \\ N}}$$

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Class - 11 - Maths Misc. Exercise (8)

Q.5 Evaluate $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$

$$\begin{aligned}
 (\sqrt{3} + \sqrt{2})^6 &= {}^6C_0 (\sqrt{3})^6 (\sqrt{2})^0 + {}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_2 (\sqrt{3})^4 (\sqrt{2})^2 + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_4 (\sqrt{3})^2 (\sqrt{2})^4 \\
 &\quad + {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5 + {}^6C_6 (\sqrt{3})^0 (\sqrt{2})^6 \\
 (\sqrt{3} - \sqrt{2})^6 &= + {}^6C_0 (\sqrt{3})^6 (\sqrt{2})^0 - {}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_2 (\sqrt{3})^4 (\sqrt{2})^2 - {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_4 (\sqrt{3})^2 (\sqrt{2})^4 \\
 &\quad - {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5 + {}^6C_6 (\sqrt{3})^0 (\sqrt{2})^6
 \end{aligned}$$

Subtract

$$\begin{aligned}
 (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 &= 2 \times \left({}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5 \right) \\
 &= 2 \cdot \left[\underset{\substack{\downarrow \\ 6 \cdot 9 \sqrt{3} \cdot \sqrt{2}}}{\cancel{{}^6C_1}} + \underset{\substack{\downarrow \\ 20 \cdot 3 \sqrt{3} \cdot 2 \sqrt{2}}}{\cancel{{}^6C_3}} + \underset{\substack{\downarrow \\ 6 \cdot \sqrt{3} \cdot 4 \sqrt{2}}}{\cancel{{}^6C_5}} \right] \\
 &= 2 \left[54 \cdot \sqrt{6} + 120 \cdot \sqrt{6} + 24 \sqrt{6} \right] \\
 &= 2 \times 198 \sqrt{6} = 396 \sqrt{6}
 \end{aligned}$$

$$\begin{array}{r}
 54 \\
 + 120 \\
 + 24 \\
 \hline
 198
 \end{array}$$

Q.6 $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4 = ?$

$\underline{4C_2 = 6}$, $\underline{4C_0 = 1}$, $\underline{4C_4 = 1}$

$$(a^2 + \sqrt{a^2 - 1})^4 = {}^4C_0 (a^2)^4 (\sqrt{a^2 - 1})^0 + {}^4C_1 (a^2)^3 (\sqrt{a^2 - 1})^1 + {}^4C_2 (a^2)^2 (\sqrt{a^2 - 1})^2 + {}^4C_3 (a^2)^1 (\sqrt{a^2 - 1})^3 + {}^4C_4 (a^2)^0 (\sqrt{a^2 - 1})^4$$

$$(a^2 - \sqrt{a^2 - 1})^4 = {}^4C_0 (a^2)^4 (\sqrt{a^2 - 1})^0 - {}^4C_1 (a^2)^3 (\sqrt{a^2 - 1})^1 + {}^4C_2 (a^2)^2 (\sqrt{a^2 - 1})^2 - {}^4C_3 (a^2)^1 (\sqrt{a^2 - 1})^3 + {}^4C_4 (a^2)^0 (\sqrt{a^2 - 1})^4$$

+

add

$$(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4 = 2 \cdot \left\{ \underline{a^8} + \underline{6 \cdot a^4 \cdot (a^2 - 1)} + \underline{(a^2 - 1)^2} \right\}$$

$$= 2 \cdot \left\{ a^8 + \underline{6 \cdot a^6} - \underline{6a^4} + \underline{a^4} - 2a^2 + 1 \right\}$$

$$= 2 \cdot \left\{ a^8 + 6 \cdot a^6 - 5 \cdot a^4 - 2a^2 + 1 \right\}$$

$$= 2 \cdot a^8 + 12 \cdot a^6 - 10 \cdot a^4 - 4 \cdot a^2 + 2 \quad \checkmark$$

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Class - 11 - Maths Misc. Ex. (8)

[Q.7] $(0.99)^5$ by only first (3) terms
(Approximate)

$$(0.99)^5 = (1 - 0.01)^5$$

$$\approx \underbrace{{}^5C_0 \cdot (1)^5 \cdot (-0.01)^0}_{1^{st}} + \underbrace{{}^5C_1 \cdot (1)^4 \cdot (-0.01)^1}_{2^{nd}} + \underbrace{{}^5C_2 \cdot (1)^3 \cdot (-0.01)^2}_{3^{rd} \text{ term (only)}} \quad \text{XX}$$

$$= 1 \cdot 1 \cdot 1 - 5 \times 1 \times 0.01 + (10 \times 1) \times (0.0001)$$

$$= \underline{1} - 0.05 + \underline{0.001}$$

$$= 1.001 - 0.05$$

$$= 0.951$$

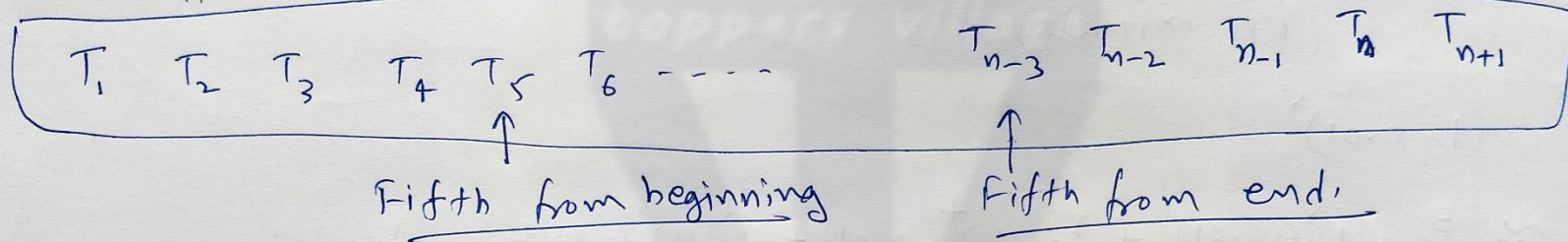
✓✓

$$\begin{array}{r} 1.001 \\ - 0.05 \\ \hline 0.951 \end{array}$$

Q. 8 $\left[\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}} \right]^n \approx (a+b)^n$ $\underline{a = (2)^{\frac{1}{4}}}$, $\underline{b = \left(\frac{1}{3}\right)^{\frac{1}{4}} = \frac{1}{3^{\frac{1}{4}}}}$

Index = power = n

No. of Terms = $n+1$



ATQ:

$$\frac{T_5}{T_{n-3}} = \frac{\sqrt{6}}{1}$$

General Term $T_{r+1} = {}^n C_r \cdot a^{n-r} \cdot b^r$
↑
one less

$$\Rightarrow \frac{{}^n C_4 \cdot a^{n-4} \cdot b^4}{{}^n C_{n-4} \cdot a^{n-(n-4)} \cdot b^{n-4}} = \sqrt{6}$$

$$\Rightarrow \frac{\left(\frac{n!}{(n-4)! \cdot 4!} \right) \times a^{n-4} \times b^4}{\left(\frac{n!}{4! \cdot (n-4)!} \right) \times a^4 \times b^{n-4}} = \sqrt{6}$$

$$\Rightarrow a^{n-8} \times b^{4-(n-4)} = \sqrt{6}$$

$$\Rightarrow a^{n-8} \times b^{8-n} = \sqrt{6}$$

$$\Rightarrow \frac{a^{n-8}}{b^{n-8}} = \sqrt{6}$$

$$\Rightarrow \frac{a^{n-8}}{b^{n-8}} = \sqrt{6}$$

$$\Rightarrow \left(\frac{a}{b} \right)^{n-8} = \sqrt{6}$$

$$\Rightarrow \left[\frac{\left(2^{\frac{1}{4}} \right)}{\left(\frac{1}{3^{\frac{1}{4}}} \right)} \right]^{n-8} = \sqrt{6}$$

$$\Rightarrow \left(2^{\frac{1}{4}} \cdot 3^{\frac{1}{4}} \right)^{n-8} = \sqrt{6}$$

$$\Rightarrow \left[(6)^{\frac{1}{4}} \right]^{n-8} = (6)^{\frac{1}{2}}$$

$$\Rightarrow 6^{\left(\frac{n-8}{4} \right)} = 6^{\left(\frac{1}{2} \right)}$$

$$\therefore \frac{n-8}{4} = \frac{1}{2} \Rightarrow n-8 = 2$$

$$\boxed{n=10} \checkmark$$

Q.9 $\left(1 + \underbrace{\frac{x}{2}}_a - \underbrace{\frac{2}{x}}_b\right)^4; x \neq 0$

$$= \underbrace{{}^4C_0 \cdot 1^4 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^0}_{\checkmark} + \underbrace{{}^4C_1 \cdot 1^3 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^1}_{\checkmark} + \underbrace{{}^4C_2 \cdot 1^2 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^2}_{\checkmark} + \underbrace{{}^4C_3 \cdot 1^1 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^3}_{\downarrow} + \underbrace{{}^4C_4 \cdot 1^0 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^4}_{\downarrow}$$

$$\begin{aligned} \left(\frac{x}{2} - \frac{2}{x}\right)^3 &= {}^3C_0 \left(\frac{x}{2}\right)^3 \left(\frac{2}{x}\right)^0 - {}^3C_1 \left(\frac{x}{2}\right)^2 \left(\frac{2}{x}\right)^1 + {}^3C_2 \left(\frac{x}{2}\right)^1 \left(\frac{2}{x}\right)^2 - {}^3C_3 \left(\frac{x}{2}\right)^0 \left(\frac{2}{x}\right)^3 \\ &= 1 \cdot \frac{x^3}{8} \cdot 1 - 3 \cdot \frac{x^2}{4} \cdot \frac{2}{x} + 3 \cdot \frac{x}{2} \cdot \frac{4}{x^2} - 1 \cdot 1 \cdot \frac{8}{x^3} \end{aligned}$$

$$= \frac{x^3}{8} - \frac{3x}{2} + \frac{6}{x} - \frac{8}{x^3}$$

$$\begin{aligned} \left(\frac{x}{2} - \frac{2}{x}\right)^4 &= {}^4C_0 \left(\frac{x}{2}\right)^4 \left(\frac{2}{x}\right)^0 - {}^4C_1 \left(\frac{x}{2}\right)^3 \left(\frac{2}{x}\right)^1 + {}^4C_2 \left(\frac{x}{2}\right)^2 \left(\frac{2}{x}\right)^2 - {}^4C_3 \left(\frac{x}{2}\right)^1 \left(\frac{2}{x}\right)^3 \\ &\quad + {}^4C_4 \left(\frac{x}{2}\right)^0 \left(\frac{2}{x}\right)^4 \end{aligned}$$

$$= 1 \cdot \frac{x^4}{16} \cdot 1 - 4 \cdot \frac{x^3}{8} \cdot \frac{2}{x} + 6 \cdot \frac{x^2}{4} \cdot \frac{4}{x^2} - 4 \cdot \frac{x}{2} \cdot \frac{8}{x^3} + 1 \cdot 1 \cdot \frac{16}{x^4}$$

$$= \frac{x^4}{16} - x^2 + 6 - \frac{16}{x^2} + \frac{16}{x^4}$$

$$\begin{aligned}
 \left(1 + \frac{x}{2} - \frac{2}{x}\right)^4 &= {}^4C_0 \cdot 1^4 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^0 + {}^4C_1 \cdot 1^3 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^1 + {}^4C_2 \cdot 1^2 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^2 + {}^4C_3 \cdot 1^1 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^3 + {}^4C_4 \cdot 1^0 \cdot \left(\frac{x}{2} - \frac{2}{x}\right)^4 \\
 &= 1 \cdot 1 \cdot 1 + 4 \cdot 1 \cdot \left(\frac{x}{2} - \frac{2}{x}\right) + 6 \cdot 1 \cdot \left(\frac{x^2}{4} + \frac{4}{x^2} - 2\right) + 4 \cdot \left(\frac{x^3}{8} - \frac{3x}{2} + \frac{6}{x} - \frac{8}{x^3}\right) \\
 &\quad + 1 \cdot 1 \cdot \left\{ \frac{x^4}{16} - x^2 + 6 - \frac{16}{x^2} + \frac{16}{x^4} \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \textcircled{1} + \textcircled{2x} - \textcircled{\frac{8}{x}} + \textcircled{\frac{3}{2}x^2} + \textcircled{\frac{24}{x^2}} - \textcircled{12} + \textcircled{\frac{x^3}{2}} - \textcircled{6x} + \textcircled{\frac{24}{x}} - \textcircled{\frac{32}{x^3}} \\
 &\quad + \textcircled{\frac{x^4}{16}} - \textcircled{x^2} + \textcircled{6} - \textcircled{\frac{16}{x^2}} + \textcircled{\frac{16}{x^4}}
 \end{aligned}$$

$$= \frac{x^4}{16} + \frac{x^3}{3} + \frac{x^2}{2} - 4x - 5 + \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4}$$

Q.10

$$\underbrace{(3x^2 - 2ax)}_A + \underbrace{3a^2}_B$$

$$= {}^3C_0 \cdot \underbrace{(3x^2 - 2ax)^3}_A \cdot (3a^2)^0 + {}^3C_1 \cdot \underbrace{(3x^2 - 2ax)^2}_A \cdot (3a^2)^1 + {}^3C_2 \cdot \underbrace{(3x^2 - 2ax)}_A \cdot (3a^2)^2 + {}^3C_3 \cdot \underbrace{(3x^2 - 2ax)^0}_A \cdot (3a^2)^3$$

$$\begin{aligned} (3x^2 - 2ax)^3 &= (3x^2)^3 - 3(3x^2)^2(2ax) + 3(3x^2)(2ax)^2 - (2ax)^3 \\ &= 27x^6 - 54a \cdot x^5 + 36a^2x^4 - 8a^3x^3 \end{aligned}$$

$$\begin{aligned} &= 1 \cdot (27x^6 - 54a \cdot x^5 + 36a^2x^4 - 8a^3x^3) \cdot 1 + 3 \cdot (9x^4 - 12a \cdot x^3 + 4a^2x^2) \cdot 3a^2 \\ &\quad + 3 \cdot (3x^2 - 2ax) \cdot (9a^4) + 1 \cdot 1 \cdot 27a^6 \end{aligned}$$

$$\begin{aligned} &= \underline{27x^6} - \underline{54a \cdot x^5} + \underline{36a^2x^4} - \underline{8a^3x^3} + \underline{81a^2x^4} - \underline{108a^3x^3} + \underline{36a^4x^2} \\ &\quad + \underline{81a^4x^2} - 54a^5x + 27a^6 \end{aligned}$$

$$= \underline{27x^6 - 54a \cdot x^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
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